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VARIABILITY OF THE SPEED OF LIGHT

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Abstract

General relativity suggests that the speed of light is influence by the gravitational potential. Several experiments illustrate variations in time attributed to the presence of the gravitational potential. These experiments can be interpreted to support the variability of the speed of light. However, this interpretation has not been definitively established or universally accepted.

Using the Schwarzschild metric, a quantification of the influence of gravity on the speed of light can be inferred. This paper proposes that the variation in the speed of light could occur most strongly in the vicinity of a massive object (e.g., in the vicinity of a massive black hole).

Keywords: General Relativity, Schwarzschild Geometry, Variability of the Speed of Light, Local and Global Reference Frames.

1.0 Introduction

Unlike the special theory of relativity, the general theory admits the possibility of the variability of the speed of light. Within the scope of general relativity, this variability is not definitively established or universally accepted.

Additional considerations involving the variability of the speed of light were provided by Santilli [(1) – (5)]. These include the importance of evaluating the local spacetime geometry and associated perturbations [1], incorporating inherent symmetries and conservation laws [2], variability of the speed of light with local physical conditions [3], arbitrary speeds for interior dynamical problems that are compatible with the abstract axioms of special relativity [4], and interior dynamics problems encountered in general relativity [5]. Santilli also established the variability of the speed of light within physical media utilizing new isomathematics [1]. Ahmar et al. [6] provide additional experimental confirmations of Santilli's assertions.

One of the key postulates of the special theory of relativity (SR) is that the speed of light c is a constant that is independent of the relative motion between inertial reference frames. A second postulate is the concept that the laws of physics are the same, and have the same form in all inertial reference frames. These postulates lead to a one to one correspondence between the metric tensor coordinates and the physical quantity (i.e., length and time)[7]. For example, the coordinate time t and proper time τ are the same.

This is illustrated by considering the coordinates utilized in special relativity in which spacetime is flat. Each inertial reference frame is distinguished from others by its relative uniform motion. There are no matter or energy perturbations in flat spacetime, but these occur in general relativity (e.g., mass distributions or sources of energy). These mass and energy sources alter the flat spacetime metric tensor that has the form:

$$g_{\mu\nu} = \mathrm{diag}[+1,-1,-1,-1] \tag{1}$$

with an associated line element

$$ds^2 = g_{\mu\nu}\, dx^{\mu}\, dx^{\nu} \tag{2}$$

where x^{μ} are the metric coordinates. Matter and energy sources create curved spacetime and fall within the scope of general relativity. For specificity, the metric of (1) is utilized instead of $g_{\mu\nu} = \mathrm{diag}[-1,+1,+1,+1]$ that is utilized in some texts including [8].

This situation is considerably more complex within the framework of general relativity that is illustrated by considering the field equations

$$G_{\mu\nu} = -\frac{8\pi G}{c^4}T_{\mu\nu} \tag{3}$$

where $G_{\mu\nu}$ is the Einstein tensor, G is the gravitational constant, and $T_{\mu\nu}$ is the energy momentum tensor [8]. The existence of mass and various source terms perturb flat spacetime and impact the postulates of special relativity. The effects of these source terms on the constancy of the speed of light (c) are the subject of this paper.

Variability of c is a controversial topic. Magueijo [9] outlines the various arguments supporting and challenges the variable c concept. [9] also describes the various theoretical arguments that lead to a variation in the speed of light within the scope of general relativity.

This paper reviews the various experiments supporting the gravitational time delay that support the variability of the speed of light, and offers additional theoretical arguments to support the variability. Using the Schwarzschild metric, the gravitational potential impact on the speed of light is quantified. It should be noted that although these assertions are speculative [9], the variability of the speed of light is an important concept that has astrophysical implications. These implications will also be reviewed.

2.0 Background

Variable speed of light has been investigated through a number of experiments that attempt to measure the gravitational effects on the concept of time. These basic concepts would affect the speed of light through the basic relationship: speed = length/time. Specific experiments that have investigated gravitational time dilation include the: (1) Hafele-Keating (HK) experiments [(10)-(12)], (2) Shapiro time delay test [(13) – (15)], (3) Gravity Probe A [16], (4) Global Positioning System [17], (5)

gravitational frequency shift (gravitational redshift) [(18) , (19)], and (6) Pound-Rebka experiment [(20), (21)]. Each of these experiments is summarized in subsequent sections. There are other experiments that also suggest temporal effects, but the aforementioned set of experiments is representative.

The aforementioned tests will be reviewed in terms of their impacts on time as affected by the strength of the gravitational field. In particular, the variability of the speed of light depends on the gravitational influence on time as noted in the aforementioned experiments. In a simplistic manner, variable speed of light c' depends on the gravitationally altered length l' and time (t') changes resulting from the influence of the gravitational potential

$$c' = \frac{l'}{t'} \tag{4}$$

The aforementioned experiments address time induced changes resulting from the gravitational potential. Length contraction within the scope of general relativity is also suggested [(22) – (24)].

This assertion of (4) is based on the assumption that the definition of speed in general relativity is essentially the same as the definitions utilized in classical mechanics and special relativity. The speed of light should have no global significance in general relativity because it depends on the length and time intervals that are well defined locally, but not globally. The lack of global definitions for length and time arise from the variability of generalized, curved Riemannian spacetime. Therefore, it may be argued that the speed of light is constant locally, but not globally with the variability induced by the gravitational potential.

These arguments are also supported by general considerations of inertial reference frames in curved spacetime. In general, there are no global inertial frames in the curved spacetime within the scope of general relativity that extend over the full spacetime domain. However, there are local inertial frames in the neighborhood of each point and in the vicinity of freely falling observers. However, there are no such global inertial reference frames. Therefore, there is an argument to be made for a distinction between local and global physical quantities including the speed of light.

Although variability of the speed of light is not a universally accepted concept, a theoretical argument is presented in this paper. This argument is based on consideration of the Schwarzschild metric, but the results can be generalized to other Riemannian geometries.

2.1 Hafele-Keating (HK) Experiment

The HK experiment was conducted in 1971 [(10) – (12)]. It is based on the combined time dilation effects caused by motion (special relativity) and the gravitational potential (general relativity). The experiment utilized atomic clocks and compared airborne and ground times.

Four cesium atomic clocks were flown on flights around the world. This was performed in eastward and westward directions to test Einstein's theory of relativity. Using flight data for each trip (e.g., altitude and velocity), the theory predicts that the airborne clocks, compared to reference clocks at the U.S. Naval Observatory should have lost 40 ± 23 nanoseconds during the eastward trip, and gained 275 ± 21 nanoseconds during the westward trip [10]. Relative to the atomic time scale of the U.S. Naval Observatory, the flying clocks lost 59 ± 10 nanoseconds during the eastward trip and gained 273 ± 7 nanoseconds during the westward trip, where the errors are the corresponding standard deviations [11].

As expected the airborne clocks exhibited time dilation caused by the motion of the east and west reference frames relative to the earth, and the differences in gravitational potential between the aircraft and ground elevations. The effects are small, but detectable. Larger gravitational fields would be expected to significantly increase the time dilation effect.

2.2 Shapiro Time Delay Test

The Shapiro time delay test [(13) – (15)] is an explicit experiment designed to provide additional confirmation of Einstein's theory of general relativity. This test measured the time delays between transmission of radar pulses directed toward Venus and Mercury. Subsequently, the detection of the reflections from these signals were determined. In accordance with the theory of general relativity, the speed of light depends

on the strength of the gravitational potential characterized by the trajectory. Shapiro notes that, these time delays should be increased by almost $2x10^{-4}$ s when the radar signals pass near a strong gravitational source (i.e., the sun). The dependence of the speed of light on the gravitational potential supports the proposed work.

Shapiro et al. [15] noted that these echoes are expected on the basis of general relativity to be retarded by solar gravity by an amount

$$\Delta t \approx \left(\frac{4r_o}{c}\right)\ln\left[\frac{(r_e + r_p + R)}{[r_e + r_p - R]}\right] \tag{5}$$

where Δt, expressed in harmonic coordinates, is the coordinate-time retardation, $r_o = 1.5$ km is the gravitational radius of the sun, c is the speed of light far from the sun, r_e is the earth-sun distance, r_p is the planet-sun distance, and R is the Earth-planet distance. The quantity Δt [15] is indicative of the magnitude and behavior of the measurable effect as predicted by general relativity. To determine if the time delay data are in agreement with this theory, Shapiro et al. [15] inserted an *ad hoc* multiplicative parameter λ on the right side of (5). A value of $\lambda = 1.0$ would indicate agreement with the predictions of general relativity. The HK experiment was first performed by Shapiro et al. in 1967 [14], and yielded the result $\lambda = 0.09 \pm 0.2$. Shapiro et al. [15] performed additional measurements with a resulting value of $\lambda = 1.01 \pm 0.02$, and this result resolved the inconsistencies in the previous preliminary analysis [14]. This experiment reinforces the validity of the time delay predicted by general relativity, and the associated variability of the speed of light.

2.3 Gravity Probe A

Gravity Probe A was a joint collaboration of the Smithsonian Astrophysical Observatory and the George C. Marshall Space Flight Center [16]. The experiment tested gravitational induced time dilation involving a frequency comparison of continuous wave microwave signals generated from hydrogen masers. These devices were located in a spacecraft at an altitude of about 10^4 km and on the surface of the earth. Agreement of the observed relativistic frequency shift with the predictions of general relativity is at the $70x10^{-6}$ level.

2.4 Global Positioning System

Ashby [17] provides a description of the Global Positioning System (GPS) and the importance of applying the concepts of general relativity to ensure that it effectively functions. The GPS uses accurate, stable atomic clocks located on satellites and on the earth's surface to provide a precise global position and time determination. These clocks exhibit both gravitational and motional frequency shifts.

The gravitational effects are defined by general relativity and the motional consequences by special relativity. These effects are sufficiently large, and must be included in the GPS design or the system would not properly function. In particular the GPS must consider a number of relativistic effects [17] including the: (1) constancy of the speed of light, (2) equivalence principle, (3) gravitational frequency shifts, (4) relativity of synchronization, (5) Sagnac effect, and (6) time dilation. In the context of the GPS, the constancy of the speed of light is applicable locally (i.e., in the vicinity of earth's weak gravitational field where spacetime is effectively flat [8]).

2.5 Gravitational Frequency Shift (Redshift)

The gravitational redshift was predicted by Einstein's general theory of relativity [18]. Consider two clocks located in differing gravitational potentials. According to the equivalence principle, the stationary clock exhibits a frequency shift [19]

$$\frac{\Delta \nu}{\nu} = \frac{\Delta U}{c^2} \tag{6}$$

where $\Delta \nu / \nu$ is the fractional frequency difference and ΔU is the gravitational potential difference between the two clocks locations. A successful measurement requires precision time measurements at the accuracy level of obtained by an atomic clock [19].

To achieve this level of precision, Shen et *al.* [19] proposed a gravitational redshift test based on frequency signals transmitted between a spacecraft and a ground station. The approach integrates one uplink signal from ground to spacecraft and two downlink signals from spacecraft to ground. The frequency shift is obtained by correlating the three frequency values.

As proposed by Shen et *al.*, using signal integration and correction models, the gravitational shift of the signals between spacecraft and ground station can be detected at a sufficiently high level of high precision. The gravitational redshift from these proposed frequency measurements provide an additional test that validates the time dilation effect.

2.6 Pound-Rebka Experiment

The Pound-Rebka experiment was designed to measure the gravitational frequency shift [(20), (21)]. This experiment measured the redshift of light traversing a gravitational field. Equivalently, the experiment is a test of the prediction of general relativity that suggests that clocks should run at different rates at different locations in a gravitational field.

Pound and Rebka outlined an experimental approach [20] to measure the gravitational redshift utilizing a ^{57}Fe photon source over a vertical distance of about 25.5 m [21]. The experiment [21] was performed to measure the effect of the gravitational potential differences at the surface of the earth and at an elevation of about 22.5m.

[21] determined a net fractional shift of $-(5.13 \pm 0.51)\times10^{-15}$. The observed shift agrees with the predicted gravitational shift of -4.92×10^{-15} for the two-way height difference utilized in the experiment.

3.0 Field Equations of General Relativity

The Einstein tensor $G_{\mu\upsilon}$ represents the geometric component of spacetime and the energy-momentum tensor $T_{\mu\upsilon}$ quantifies its physical matter and energy properties [8] as given in (3). In (3), $T_{\mu\upsilon}$ provides a source term for general relativity which is no longer restricted to the simplifying assumptions of special relativity. In particular, spacetime is no longer restricted to be flat and curvature is permitted. In curved spacetime, the metric tensor coordinates do not necessarily represent the physical coordinates. These considerations are addressed in subsequent discussion.

4.0 Schwarzschild Geometry

Differences between metric coordinates and physical coordinates are illustrated by considering the Schwarzschild geometry. The

Schwarzschild spacetime geometry in spherical coordinates (t, r, θ, φ) [8] is

$$g_{\mu\nu} = \text{diag}\left[1-\frac{2GM}{c^2 r}, -\left(1-\frac{2GM}{c^2 r}\right)^{-1}, -r^2, -r^2 \text{Sin}^2(\theta)\right] \tag{7}$$

The Schwarzschild metric is selected because there is no temporal variable, and it is time independent and spherically symmetric. This metric describes the geometry of spacetime, exterior to a static, spherically symmetric, nonrotating, and electrically uncharged body of mass M surrounded by empty space. The mass M does not vary in space or time by either translation or rotation. There is no change in its mass distribution or internal density distribution. The mass distribution has spherical symmetry, and is a function of the radial coordinate r with no dependency on θ or φ.

The condition of empty space implies the metric is the vacuum solution with $T_{\mu\nu} = 0$. Therefore, the field equation (3) reduces to

$$G_{\mu\nu} = 0 \tag{8}$$

5.0 Relationship between Coordinate Time and Proper Time

The Schwarzschild metric is utilized to review the properties of time within the context of general relativity. To illustrate the temporal characteristics, consider a stationary frame in the Schwarzschild spacetime, and events that only vary in time (i.e., dr = d θ = dφ = 0). Accordingly,

$$ds^2 = d\tau^2 = \left(1 - \frac{2GM}{c^2 r}\right) dt^2 \tag{9}$$

where t is the coordinate time and τ is the proper time. The proper time interval $d\tau$ is shorter than the coordinate time interval dt by a factor of $\left(1 - \frac{2GM}{c^2 r}\right)^{1/2}$. Coordinate time and proper time intervals are only equal

when $r \rightarrow \infty$. If there are two stationary reference frames K^1 at r_1 and K^2 at r_2 with $r_1 < r_2$

$$d\tau_1 = \left(1 - \frac{2GM}{c^2 r_1}\right)^{1/2} dt \qquad (10)$$

$$d\tau_2 = \left(1 - \frac{2GM}{c^2 r_2}\right)^{1/2} dt \qquad (11)$$

(10) and (11) suggest that $d\tau_1 < d\tau_2$ because $r_1 < r_2$. The result is significant because the proper time varies with distance from the mass M. This effect does not occur in special relativity. In particular, (10) and (11) suggest time runs slower closer to the source of gravity (i.e., the mass M) where the gravitational field is larger. The reader should note that the coordinate time t is not subscripted because the coordinate time is common to all frames of reference since it is a global coordinate. However, τ varies locally and is not a global coordinate.

6.0 Relationship between Spatial Coordinates and Proper Time

In a discussion similar to that of Section 5.0, consider a stationary frame at a specified distance from the mass M in the Schwarzschild spacetime, and consider events with no time variation ($dt = 0$). With this limitation,

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \mathrm{Sin}^2\theta d\varphi^2 \qquad (12)$$

The discussion is simplified by considering the angular and radial components separately. For the angular component, $dr = 0$ and

$$ds^2 = r^2 d\theta^2 + r^2 \mathrm{Sin}^2\theta d\varphi^2 \qquad (13)$$

which is the same quadratic form in 3 dimensional spherical coordinates on a sphere of constant r. This suggests that gravity has no direct effect on

the circumferential component of length. Therefore, the constant r surfaces are similar to their flat space spherical coordinates. The results are more complex for the radial component (i.e., $d\theta = d\varphi = 0$)

$$ds = \left(1 - \frac{2GM}{c^2 r}\right)^{-1/2} dr \tag{14}$$

If two stationary frames K^1 at r_1 and K^2 at r_2 with $r_1 < r_2$ are considered, and the discussion is limited to the radial component only

$$ds_1 = \left(1 - \frac{2GM}{c^2 r_1}\right)^{-1/2} dr \tag{15}$$

$$ds_2 = \left(1 - \frac{2GM}{c^2 r_2}\right)^{-1/2} dr \tag{16}$$

(15) and (16) suggest $ds_1 > ds_2$ because $r_1 < r_2$.

In a manner similar to the discussion of Section 5.0, dr is not subscripted because the radial coordinate is common to all reference frames since it is a global parameter. The proper length varies with the distance with $ds_1 > ds_2$ for $r_1 < r_2$. From a theoretical perspective, length is contracted further from the source of gravity where the field is weaker.

In special relativity, length is contracted with respect to two global, inertial reference frames in relative motion with respect to each other. In particular, length contraction in special relativity is dependent upon the velocity difference between the reference frames. In general relativity length contraction is caused by gravity and is a function of the spacetime location (i.e., the local metric characteristics of spacetime).

7.0 Local and Global Reference Frames

The previous sections address general considerations for the behavior of length and time in local and global reference frames. Within the context of special relativity, the constant speed of light is an inherent

postulate. This speed is constant both locally as well as globally. Inertial reference frames ensure that the speed of light is constant locally as well as globally in flat spacetime.

The question of reference frames is also an important consideration in general relativity. In general relativity, there are no global inertial reference frames in curved spacetime. However, there are local inertial frames surrounding each spacetime location. These local inertial reference frames also exist near world lines of freely falling observers. However, similar global frames do not exist in general relativity. This suggests that the postulates of special relativity will be observed locally in general relativity, but not globally. As a result, the speed of light will be constant locally in general relativity, but not globally. This assertion is addressed in more detail in subsequent discussion.

8.0 Implications for the Constancy of the Speed of Light

The basic definition of velocity (i.e., dx^i/dt in a coordinate representation or $ds/d\tau$ in a physical definition) is readily defined within the context of classical physics and special relativity. However, this interpretation is less obvious within general relativity in a strong gravitational field.

In special relativity, velocity and the speed of light are global quantities that are supported by Einstein's postulates. However, in general relativity length and time have no global significance. Length and time are constant locally, but not globally. Specifically, the speed of light is readily defined in special relativity since it is a global quantity. In general relativity, the definition is more complex

$$c' = \frac{ds_i}{d\tau_i} \qquad (17)$$

where c' is the local speed of light that is constant locally, but not globally. (17) is justified by the distinction between local and global velocity, and is consistent with (4) as well as the previous discussion regarding physical quantities in local and global reference frames.

Using (10), (15), and (17), the local speed of light can be quantified in terms of the Schwarzschild metric

$$c' = \frac{\left(1-\dfrac{2GM}{c^2 r_i}\right)^{-1/2} dr}{\left(1-\dfrac{2GM}{c^2 r_i}\right)^{1/2} dt} = \frac{1}{\left(1-\dfrac{2GM}{c^2 r_i}\right)} c \qquad (18)$$

where $c = dr/dt$ is the global speed of light as defined in the special theory of relativity. (18) suggests that the variation in the speed of light depends on the factor $(1 - 2GM/c^2 r_i)^{-1}$. A test of the validity of the local nature of the speed of light in general relativity would depend on this factor. This could be most clearly observed for large values of M and/or small values of r_i. For example, measurements close to a massive black hole could reveal the postulated variation in c, but these measurements are beyond current scientific capabilities.

These and similar measurements would provide a definitive resolution to the possibility of the local variability of c within the scope of general relativity. Until such measurements are forthcoming, the proposed variation must be considered as speculative.

The conditions of (18) could also be met during a big bang/big crunch cycle. Variation in c would explore the initial time post big bang/big crunch including the concept of cosmic inflation. The variability of c would be integral to various theories involving this initial time period.

Inflationary cosmology within the context of general relativity is addressed in (22) and (23). In particular, inflationary cosmology is believed to explain the isotropy, large scale homogeneity, and flatness of spacetime. Additionally, general relativity predicts the deviations from homogeneity of our universe.

[25] suggests that these arguments are not the only option to explain these features of the universe. Moffat proposed a model in which local Lorentz invariance is spontaneously broken in the very early universe, and in this epoch the speed of light undergoes a first or second order phase transition to a value approximately 30 orders of magnitude smaller, corresponding to the presently measured speed of light. [25] also argues that there are additional attractive features of a variable speed of light theory compared to standard inflationary theory, and that it provides an alternative cosmology with potentially different predictions.

Moffat [26] also argues that a representative cosmology is obtained in which the causal mechanism of generating primordial perturbations is achieved by varying the speed of light in a primordial epoch. This yields an alternative to inflation for explaining the formation of the cosmic microwave background and the large scale structure of the universe. Moffat's arguments provide additional motivation for studies of the variability of the speed of light.

9.0 Conclusions

The variability of the speed of light is an open theoretical issue that requires experimental verification. Gravitational time delay has been observed, but relating this observation to the variability in the speed of light depends upon defining velocity in terms of local and global reference frames. Within the context of the Schwarzschild metric, the variability of the speed of light is inferred from a consideration of local reference frames. The degree of variability depends on the strength of the gravitational potential generated by a massive object. Variable speed of light has cosmological implications including the viability of cosmic inflation as well as the evolution of the universe from the initial initiating event.

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